UDC 532.536

TURBULENT BOUNDARY LAYER IN A SUPERSONIC DIFFUSER

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Inzhenero-Fizicheskii Zhurnal, Vol. 9, No. 1, p. 3-8, 1965

The one-parameter method of calculating the turbulent boundary layer of a compressible gas with positive axial pressure gradient is analyzed. Theory is compared with experiment. One of the advantages of this method is its simplicity.

The approximate one-parameter method of calculating the supersonic turbulent boundary layer in the region of a diffuser is based on the limit laws of Kutateladze and Leontev [3], according to which the momentum equation for flow of a compressible gas is

$$\frac{d\,\mathrm{Re}^{**}}{dx} + \frac{\mathrm{Re}^{**}}{w_0}\frac{dw_0}{dx}(1 + H_{\mathrm{cr}}) = \mathrm{Re}_L\,\psi_{\mathrm{T}}\frac{C_{f_0}}{2}, \tag{1}$$

where

$$\mathrm{Re}^{**} = \frac{w_0\,\delta^{**}\rho_0}{\mu_{00}}, \quad \mathrm{Re}_L = \frac{w_0\,\rho_0\,L}{\mu_{00}},$$

$$\delta^{**} = \int_0^{\delta} \frac{\rho w}{\rho_0 w_0}\left(1 - \frac{w}{w_0}\right)dy, \quad \delta^{*} = \int_0^{\delta}\left(1 - \frac{\rho w}{\rho_0 w_0}\right)dy.$$

Equation (1) was obtained from the usual form of the momentum equation by substituting the expression

$$C_f = \psi_f \psi_{\mathrm{T}} C_{f_0} \tag{2}$$

and then linearizing as in the Loitsyanskii-Buri method. Here ψ_{T} is the relative change in the friction coefficient taking into account the compressibility and the fact that the process is nonisothermal; ψ_f is the change taking into account the axial pressure gradient. It should be noted that (2) is based on the assumption that the functions ψ_{T} and ψ_f have separate effects on the friction coefficient. There is no direct experimental confirmation of this assumption, and one can only point out that, for example, separate allowance for the nonisothermal effect and degree of injection on a permeable plate has proved very fruitful and the validity of this procedure has been confirmed theoretically [4].

Because the value of the shape factor H_{cr} enters into (1), this equation becomes more accurate in the region close to separation. It is also sufficiently accurate in the region remote from the separation point, since the second term is then generally small.

Values of ψ_{T} and H_{cr} have been found theoretically and are given in [3]. According to relations approximating the theoretical solutions

$$H_{\mathrm{cr}} = 2.41\Psi^{*} + 1.38\,\Delta\Psi - 0.53, \tag{3}$$

$$\psi_{\mathrm{T}} = \left(\frac{2}{\sqrt{\overline{\Psi}} + 1}\right)^2 \left[\frac{\operatorname{arctg}\sqrt{r(k-1)/2}\,\mathrm{M}}{\sqrt{r(k-1)/2}\,\mathrm{M}}\right]^2. \tag{4}$$

Here

$$\Psi^{*} = \frac{T_{\mathrm{w}}^{*}}{T_0} = 1 + r\frac{k-1}{2}\mathrm{M}^2,$$

$$\Delta\Psi = \Psi - \Psi^{*} \quad T_{\mathrm{w}}/T_0 - T_{\mathrm{w}}^{*}/T_0,$$

$$\overline{\Psi} = \frac{T_{\mathrm{w}}^{*}}{T_{\mathrm{w}}}.$$

Formula (3) has been confirmed by experiments for isothermal conditions [5], and (4) for zero-gradient flow [4, 6].

Expressing C_{f_0} as

$$C_{f_0} = B\,(\mathrm{Re}^{**})^{-m}\,(\mu_0/\mu_{00})^m = 0.0252\,(\mathrm{Re}^{**})^{-0.25}\,(\mu_0/\mu_{00})^{0.25}, \tag{5}$$

we obtain the integral of (1)

$$\mathrm{Re}^{**} = \frac{1}{\exp A}\left[\frac{(1+m)B}{2}\int\limits_0^{\overline{X}} \psi_{\mathrm{T}}\,\mathrm{Re}_L \exp\left[(1+m)A\right]\left(\frac{\mu_0}{\mu_{00}}\right)^{0,25} d\overline{X} + c\right]^{\frac{1}{1+m}},$$

$$A = \int (1 + H_{\mathrm{cr}})\,\frac{dw_0}{w_0}. \tag{6}$$

The dynamic turbulent boundary layer in a supersonic two-dimensional diffuser has been investigated in tests by McLafferty and Barber [1]. The pressure in the diffuser increased smoothly. The following data, given in [1] were used in a calculation based on (6): T_{00} = 388°C, diffuser wall thermally insulated, $p_{00} = 0.98 \cdot 10^5$ n/m^2 when x = 0, δ^{**} = = 0.406 mm (see Fig. 4).

Values of the Mach number along the diffuser for x = x/L = 0; 0.2; 0.4; 0.6; 0.8; 1.0 were, respectively, 3.01; 3.00; 2.68; 2.32; 2.24; 1.90.

As may be seen from the figure, the agreement between the experimental and theoretical data on momentum thickness is quite satisfactory. It should be noted, incidentally, that calculation of δ^{**}, putting $H_{cr} = H_0 = f$ (M) and ψ_T = 1 in (1) also gives results close to the experimental value, which is attributable to mutual compensation of the assumptions $H_{cr} = H_0$ and $\psi_T = 1$.

There is also practical interest in determining the displacement thickness δ^*, which may be found from a knowledge of δ^{**} and H. The value of H for gradient flow of an incompressible fluid may be found using the relation $\tilde{H} = f(\tilde{f})$, obtained from a computer solution of the corresponding system of equations [4]. This system contains, in particular, equations describing the velocity distribution in the boundary layer, the stability conditions, etc. The relation $\tilde{H} = f(\tilde{f})$ for an incompressible fluid and $\mathrm{Re}^{**} \to \infty$ is shown in Fig. 1 (1), where

$$\tilde{H} = \frac{H}{H_{\mathrm{cr}}};\quad \tilde{f} = \frac{f}{f_{\mathrm{cr}}};\quad f = \frac{\delta^{**}}{w_0}\frac{dw_0}{dx};$$

$$f_{\mathrm{cr}_0} = 0.01 - 0.044\,(\mathrm{Re}^{**})^{-0.3}.$$

The influence of compressibility on the relation $\tilde{H} = f(\tilde{f})$ in the first approximation may be found as follows. When $\tilde{f}$ = 1; $\tilde{H}$ = 1 for any value of M; by definition, here $\tilde{f} = f/f_{cr}$; $f_{cr} = [0.01 - 0.044\,(\mathrm{Re}^{**})^{-0.3}]\,\Psi^{*\,-1.53}$, i.e., in accordance with [4], the influence of compressibility on f_{cr} is taken into account. In order to find $\tilde{H}$ when $\tilde{f}$ = 0 (i.e., for the case of flow over a plate), we examine successively the quantities H_{cr} and H.

Values of H_{cr} are found according to (3). The value of H for various M numbers may be determined from the expressions for δ^* and δ^{**}, if the law of variation of velocity and density in the boundary layer is given. In our calculations we assumed a "one-seventh" law of velocity variation in the boundary layer, and the Crocco equation of density variation.

When $r = 1$ and $\dfrac{\partial p}{\partial y} = 0$

$$\frac{\rho}{\rho_0} = \frac{T_0}{T};\quad \frac{T}{T_0} = \Psi^* - (\Psi^* - 1)\,\omega^2.$$

Strictly speaking, the exponent n in the velocity distribution law is a function of the M number, but variation of n may be neglected at medium values of M, as shown by experiment [7, 8]. The assumptions made are also confirmed by the graph in Fig. 2, which shows the experimental dependence of $H/H_{n=1/7}$ on Re^{**} and M(H is the experimental value of the shape factor, and $H_{n=1/7}$ the theoretical). This graph shows that the ratio

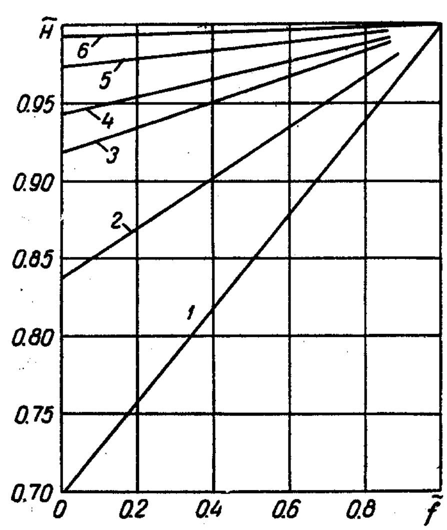

Fig. 1. Dependence of shape factor $\tilde{H}$ on separation parameter $\tilde{f}$ for $\mathrm{Re}^{**} \to \infty$ at various M numbers: 1) M = 0; 2) M = 2; 3) M = 4; 4) M = 6; 5) M = 8; 6) M = 10.

$H/H_{n=1/7}$ is quite close to unity, and no differentiation with respect to M number is seen. This gives a sure basis for using the parameter $\tilde{H}_{n=1/7}$ to determine $\tilde{H}$ when $\tilde{f} = 0$ at various M numbers, as shown in Fig. 1. For $0 < f < 1$ the values of H in the first approximation may be found from a linear approximation in terms of the two sufficiently reliable values of $\tilde{H}$ when $\tilde{f} = 0$ and $\tilde{f} = 1$, by analogy with the relation $\tilde{H} = f(\tilde{f})$ at M = 0, where it is close to linear.

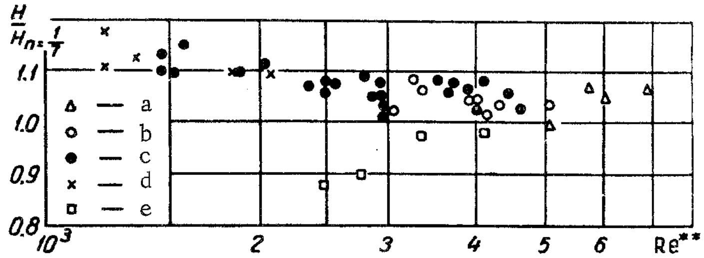

Fig. 2. Effect of Re** and M on shape factor H for flow of a liquid along a plate according to the data of [1]: a) M = 2; b) 2.5; c) 3.0; d) 3.5 (from the data of [14]); e) 5.8.

Values of H for the experimental conditions of [1] were found from (6) and Fig. 1. Figure 3 shows a comparison of the experimental and theoretical data. It can be seen from the graphs that the proposed method of determining displacement thickness gives quite acceptable results.

It should be borne in mind that the results obtained in calculating δ^{**} still do not give sufficient confirmation of assumption (2), because of the comparatively weak influence of functions ψ_T and ψ_f on Re**. The validity of relation (2) may be verified only by comparing experimental and calculated values of C_f. Unfortunately, we are not aware of the existence of experimental data on direct measurements of the friction coefficient in a flow of compressible gas in a diffuser.

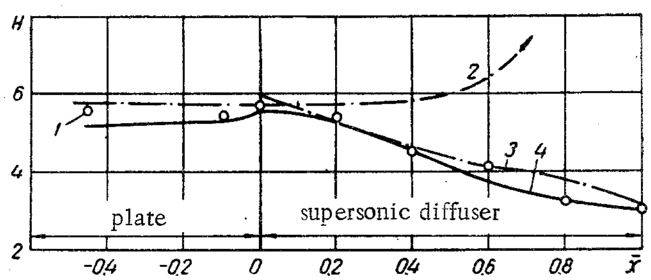

Fig. 3. Variation of H in a supersonic diffuser according to theory and experiment: 1) experiment [1]; 2) theory [13]; 3) theory [1]; 4) present method.

The method of determining local values of the friction coefficients from the logarithmic part of the velocity profile in the boundary layer in the coordinates $w/w_0\sqrt{C_f/2}$; $\log(w\sqrt{C_f/2}/\nu_w)$ has given good results for gradient flow of an incompressible fluid in the presence of cross flow [9, 10, 11]. In [7, 8] it was noted that the logarithmic part of the velocity profile is preserved for flow of a compressible gas along a flat plate, with the flow parameters ρ, μ referred to wall temperature.

We have attempted to determine the values of C_f for flow of a compressible gas in a diffuser, using the velocity profile in the boundary layer given in [1]. The calculations showed that the logarithmic part of the profile is preserved in these conditions also.

The velocity distribution in the turbulent boundary layer of a compressible gas is more rigorously determined by the "logarithmic sine" law, can also be used to find values of C_f, if one knows the empirical constants of turbulence $\varkappa$, the thickness of the viscous sublayer η_1, and the velocity at its edge ϕ_1.

According to [8, 12], for medium values of M, we may assume that $\varkappa = 0.41$, $\eta_1 = \varphi_1 = 11.6 - 11.9$.

Values of C_f found in this way are compared in Fig. 4 with calculations based on (1), (2), and (5). It should be stressed that the results of calculating C_f by the one-parameter method are strongly dependent on the variation of the axial pressure gradient. Taking this into consideration and allowing for some error in the measurement of the velocity at the outer edge of the boundary layer in [1] we may regard the agreement of the calculated and experimental C_f values as quite satisfactory. However, additional experimental material is required for final confirmation of the validity of this method of calculating the turbulent boundary layer in a flow of compressible gas in the region of a diffuser.

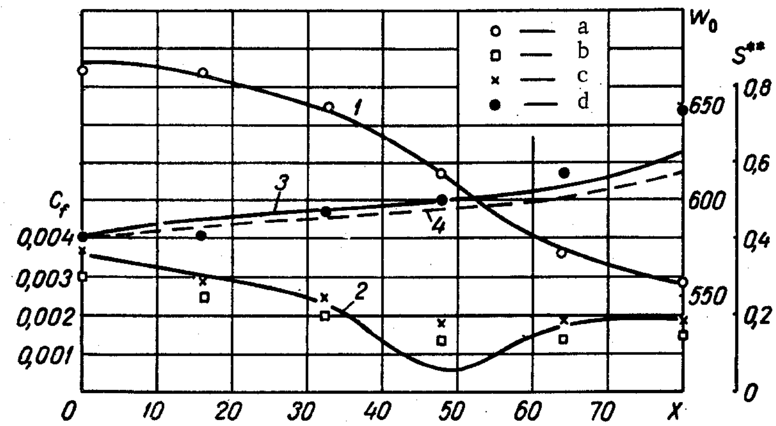

Fig. 4. Comparison of experimental and theoretical values of C_f and δ^{**}: a) experimental values of w_0 [1]; 1) law of variation of $w_0 = f(x)$ for calculation of C_f and δ^{**}; b, c) values of C_f found from the velocity profiles reduced in accordance with the "logarithmic sine" law and the universal logarithmic law; 2) theoretical values of C_f according to (1), (2), and (5); d) experimental values of δ^{**} from [1]; 3) values of δ^{**} according to (6); 4) values of δ^{**} according to (6) for $\psi_T = 1$ and $H_{cr} = H_0 = f(M)$.

NOTATION

ρ_0, w_0 and ρ, w—density and velocity at outer edge of boundary layer and in boundary layer; δ^{**}—momentum thickness; L—characteristic length $\overline{x} = x/L$; H_{cr}—value of shape factor $H = \delta^*/\delta^{**}$ at separation point; C_{f_0}—local friction coefficient on plate under isothermal conditions; ω—relative velocity in boundary layer; p_{00}—stagnation pressure; M—Mach number; r—recovery factor; k—adiabatic exponent; T_w, T_{00}, and T_0—wall, stagnation, and thermodynamic temperature; μ_{00} and μ_0—dynamic viscosity at stagnation and thermodynamic temperatures; C_f—local value of friction coefficient; Ψ^*—kinetic temperature factor; Ψ—temperature factor; H_0—value of parameter H for flow over a plate; f_{cro}—value of shape factor f at separation point. Subscripts: "0"—parameters at outer edge of boundary layer; "00"—parameters under stagnation conditions.

REFERENCES

1. G. H. McLafferty and R. E. Barber, J. Aerospace Sci., 29, no. 1, 1962.
2. J. F. Stroud and D. M. Coleman, Raketnoya tekhnologiya, no. 5, 96, 1962.
3. S. S. Kutateladze and A. I. Leontev, Turbulent Boundary Layer of a Compressible Gas [in Russian], Izd. SO AN SSSR, Novosibirsk, 1962.
4. S. S. Kutateladze (ed.), Heat and Mass Transfer and Friction in a Turbulent Boundary Layer [in Russian], Izd. SO AN SSSR, Novosibirsk, 1964.
5. Ioshimasa Furuya, Memoirs of the Faculty of Engineering, Nagoya University, Nagoya, Japan, 10, no. 1, 1958.
6. D. B. Spalding and S. W. Chi, J. Fluid Mechan., 18, no. 1, 1964.
7. F. K. Hill, Voprosy raketnoi tekhniki, no. 1, 1957.
8. R. K. Lobb, E. M. Winkler, and J. Persh, Voprosy raketnoi tekhniki, no. 5, 1955.
9. F. H. Clauser, JAS, 21, no. 2, 1954.
10. B. M. Leadon and E. R. Bartle, J. Aerospace Sci., 27, no. 3, 1960.
11. P. N. Romanenko and A. I. Leontev, Proc. of the First Inter-University Conference [in Russian], Moscow, 139, 1961.
12. L. M. Zysina-Molozhen and I. N. Soskova, Heat and Mass Transfer III [in Russian], GEI, 1963.
13. E. Reshotko and M. Tucker, NACA Tech Reports, no. 4154, Dec. 1957.
14. R. H. Korkegi, J. Aeronautical Sci., 23, no. 2, 1956.

30 October 1964

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